## The Day Through the Seasons

## Rising and Setting of the Sun: Seasons and Latitude

The azimuths where an object at a declination $\delta$ will rise and set, and the length of time that object will be above the horizon as seen by an observer at a latitude $\lambda$ are given by:

$$
A_{\text {rise }}=\cos ^{-1}\left(\frac{\sin \delta}{\cos \lambda}\right) \text { degrees } \quad A_{\text {set }}=360-A_{\text {rise }} \text { degrees } \quad \Delta t=\frac{2}{15} \cos ^{-1}(-\tan \lambda \tan \delta) \text { hours }
$$

Using these, find the rising and setting azimuths and the length of the day for the following locations and dates, using the declination of the sun given by the analemma: ${ }^{30}$

| Date | $\delta \odot$ | Canton, New York$\left(\lambda=44^{\circ} 36^{\prime} N\right)$ |  |  | Quito, Equador$\left(\lambda=0^{\circ} \mathrm{N}\right)$ |  |  | Igloolik, Nunavut, Canada$\left(\lambda=69^{\circ} 24^{\prime} \mathrm{N}\right)$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $A_{\text {rise }}$ | $\mathrm{A}_{\text {set }}$ | $\Delta t_{\text {day }}$ | $A_{\text {rise }}$ | $\mathrm{A}_{\text {set }}$ | $\Delta t_{\text {day }}$ | $A_{\text {rise }}$ | $\mathrm{A}_{\text {set }}$ | $\Delta t_{\text {day }}$ |
| Feb. 12 | $-14^{\circ}$ | $109.9{ }^{\circ}$ | $250.1^{\circ}$ | 10.1 h | $104.0^{\circ}$ | $256.0^{\circ}$ | 12 h | $133.4^{\circ}$ | $226.6^{\circ}$ | 6.46 h |
| Dec. 25 | $-23.5^{\circ}$ | $124.1^{\circ}$ | $235.9^{\circ}$ | 8.6 h | $113.5^{\circ}$ | $246.5^{\circ}$ | 12 h | Never Rises |  | O h |
| Jun. 25 | $+23.5^{\circ}$ | $55.9{ }^{\circ}$ | $304.1{ }^{\circ}$ | 15.4 h | $66.5^{\circ}$ | $293.5^{\circ}$ | 12 h | Never Sets |  | 24 h |

## VARIATION OF THE SOLAR DAY: THE ANALEMMA

If you mark the position of the tip of a sundial's shadow at noon each day of the year, it will scribe the shape known as the ANALEMMA. This shape describes the difference between mean solar noon (or clock noon) and solar noon. The analemma's shape is due to both the tilt of the Earth's axis, and the elliptical shape of the Earth's orbit. Check out www.analemma.com for details.

This is used to find the declination of the Sun and the time by which the Sun is early or late for clock noon (mean solar noon ... not accounting for the observer's position within the time zone). ${ }^{20}$


LATE: Sol is to the east of the meridian at clock noon (Sun has not yet arrived at noon) EARLY: Sol is to the west of the meridian at clock noon (Sun has already passed noon).

TAKE POSITION TO THE CLOSEST $\frac{1}{2}$ DEGREE, TIME TO THE CLOSEST $\frac{1}{2}$ MINUTE.

| Date | Declination of <br> Sun | Sol's position in degrees <br> East or West of clock noon | Minutes by which Sol will be <br> early or late for clock noon | Clock time of <br> solar noon |
| :--- | :---: | :---: | :---: | :---: |
| February 12 | $-14^{\circ}$ | $3.5^{\circ}$ East | 14 min Late | $12: 14 \mathrm{pm}$ |
| June 25 | $+23.5^{\circ}$ | $0.5^{\circ}$ East | 2 min Late | $12: 02 \mathrm{pm}$ |
| September 14 | $+4^{\circ}$ | $1^{\circ}$ West | 4 min Early | $11: 56 \mathrm{am}$ |
| December 25 | $-23.5^{\circ}$ | $0^{\circ}$ | 0 min | $12: 00: 00 \mathrm{pm}$ |
| Your Birthday <br> Date: $9 / 7$ | $+6^{\circ}$ | $3.5^{\circ}$ East | 2 min Early | $11: 58 \mathrm{am}$ |

Phys 102 Astronomy

$$
\begin{aligned}
& { }_{2} \text { min }^{\text {min }} 2 \text { min } \\
& 4 \text { min } 2 \text { min } \\
& \text { tim: in minutes }
\end{aligned}
$$



